Video Notes: Basic Simplifications and Common Algebra Errors

Terms vs. Factor errors

Many properties apply only to terms or only to factors. Be clear on which is which.

(1) $(ab)^n = a^n b^n$ but $(a+b)^n \neq a^n + b^n$

powers do not "distribute over addition"

(2) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ but $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

cannot "take root term by term"

(3) $\frac{3a^{-2}b}{c} = \frac{3b}{a^2c}$ but $\frac{3a^{-2}+b}{c} \neq \frac{3+b}{a^2c}$

factors "jump fraction bar" to change sign of exponent terms do not

(4) $\frac{2xy}{5x} = \frac{2\cancel{x}y}{5\cancel{x}} = \frac{2y}{5}$ but $\frac{2x+y}{5x} \neq \frac{2\cancel{x}+y}{5\cancel{x}}$

factors divide out terms do not "cancel"

(5) 3(x+y)=3x+3y $10(0.2x) \neq 10(0.2) \bullet 10x$

"multiplication distributes over addition" but mult does not "distribute over mult"

instead, the associative law applies

$10(0.2x) = (10 \bullet 0.2)x = 2x$

Missing or "invisible" parenthesis

(6) $(-3)^2 = (-3)(-3) = 9$ is not the same as -3^2

$-3^2 = -(3)^2 = -(3 \bullet 3) = -9$

(7) $(5x)^{-2} = \frac{1}{(5x)^2} = \frac{1}{25x^2}$ is not the same as $5x^{-2}$

$5x^{-2} = 5 \bullet x^{-2} = 5 \bullet \frac{1}{x^2} = \frac{5}{x^2}$

(8) (x+2)(x+1) is not the same as x+2(x+1)

(9) 3x-(x+1) is not the same as 3x-x+1

Square roots and Absolute Values

(10) $\sqrt{16} = 4$ not ± 4

(11) If $x^2 = 49$ then x = $\pm\sqrt{49} = \pm 7$ not just 7.

(12) $\sqrt{x^2} = |x|$ not just x

Worksheet: Basic Simplifications and Common Algebra Errors

Answer True or False. If the answer is false, what is the correct simplification

1) $\sqrt{x^2+16}$ simplifies to $x+4$ ________________

2) $(\sqrt{x}+3)^2 = x+6\sqrt{x}+9$ ______________________________________

3) $\dfrac{x^2y-x}{x^2(x+4)}$ simplifies to $\dfrac{\cancel{x^2}y-x}{\cancel{x^2}(x+4)} = \dfrac{y-x}{x+4}$ _____________________________________

4) $\sqrt{25} = \pm 5$ ______________________________________

5) $(x+2)^3$ simplifies to x^3+8 ______________________________________

6) If $x^2=32$ then $x=4\sqrt{2}$ ___________________________________

7) $7x^{-2}y$ simplifies to $\dfrac{7y}{x^2}$ ____

8) $\sqrt{(x-2)^2}$ ____________________________________

9) $\dfrac{4y^{-2}-x}{y}$ simplifes to $\dfrac{4-x}{y^3}$ __________________________________

10) $\sqrt{a^2+9a^4}$ simplifies to $a+3a^2$ _____________________________________

Video Notes: Algebraic Simplifications: Factoring Rational Exponents

(1) Factor:

$3x^5 - 12x^3$

$\frac{1}{2}x - 4$

$4x^{\frac{-2}{3}} - 8x^{\frac{1}{3}}$

$2(2x+5)(2)\sqrt{4-x} - \frac{1}{2}\left(2x+5\right)^2 (4-x)^{-\frac{1}{2}}$

Worksheet: Algebraic Simplifications: Factoring Rational Exponents

(1) Factor: (factor out fractional coefficients also)

(a) $\frac{2}{3}x^3 - 4x^2$

(c) $64x^{\frac{2}{3}} - 100x^{\frac{5}{3}}$

Ans: $\frac{2}{3}x^2(x-6)$

Ans: $4x^{2/3}(16-25x)$

(b) $12x^{\frac{-3}{4}} - 8x^{\frac{1}{4}}$

(d) $-\frac{1}{2}(3x)\left(1-x^2\right)^{-\frac{3}{2}}(-2x) + 3\left(1-x^2\right)^{-\frac{1}{2}}$

Ans: $\frac{4(3-2x)}{x^{3/4}}$

Ans: $\frac{3}{(1-x^2)^{3/2}}$

Video Notes: Complex Fractions

Simplify

$$\frac{2\sqrt{1+x}-\frac{x}{\sqrt{1+x}}}{1+x}$$

$$\frac{x(8x-1)\left(x^2+5\right)^{-\frac{1}{2}}-8\left(x^2+5\right)^{\frac{1}{2}}}{(8x-1)^2}$$

Worksheet: Complex Fractions

Simplify: You might try each in both of the ways shown on the video.

(a) $$\frac{2x\sqrt{x+3}-\dfrac{x^2}{\sqrt{x+3}}}{x+3}$$

(b) $$\frac{\frac{1}{3}(x^2+1)x^{-\frac{2}{3}}-2x^{\frac{4}{3}}}{(x^2+1)^2}$$

Ans: $$\frac{x^2+6x}{(x+3)^{3/2}}$$

Ans: $$\frac{1-5x^2}{3x^{2/3}(x^2+1)^2}$$

Video Notes: Nonlinear Inequalities / Sign Charts

Solve $x^2 - x < 6$

$$\frac{x-2}{(x+1)^2(x-4)} \geq 0$$

$2\sin^2 x - \sin x \leq 0; \quad 0 \leq x \leq 2\pi$

Worksheet: Nonlinear Inequalities / Sign Charts

(1) Solve $\frac{x-2}{x^2-16} \geq 0$

(2) Find the domain f(x)= $\sqrt{3x^2-6x}$

Ans: $(-4,2] \cup (4,\infty)$

Ans: $(-\infty,0] \cup [2,\infty)$

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(3) $\cos^2 x - \cos x \leq 0; \quad 0 \leq x \leq 2\pi$

Ans: $[0,\frac{\pi}{2}] \cup [\frac{3\pi}{2}, 2\pi]$

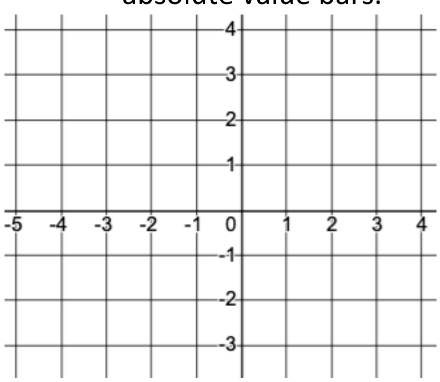
Video Notes: Working with Absolute Value-Notes

Algebraic Definition of Absolute Value: $\begin{cases} \_\_\_\_\_\_\_ \; if \; \_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_ \; if \; \_\_\_\_\_\_\_\_\_ \end{cases}$

Note also: $\sqrt{x^2} =$ ______

Often in Calculus, we will need to "remove the bars", that is write absolute value expressions as piecewise defined functions.

EX: Graph f(x)= $\frac{|x|}{x}$ by first writing it as a piecewise defined function without absolute value bars.

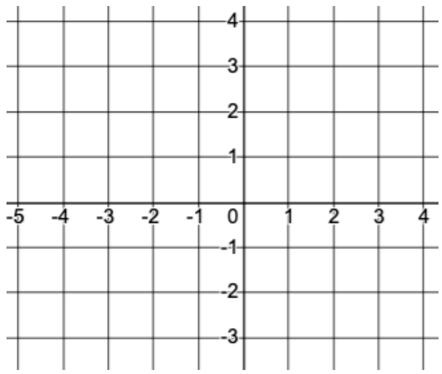


EX: Rewrite the function $f(x) = |1-4x|$ as a piecewise function with no bars.

EX: Rewrite the function $f(x) = |x^2 - x - 12|$ as a piecewise function with no bars.

Worksheet: Working with Absolute Value

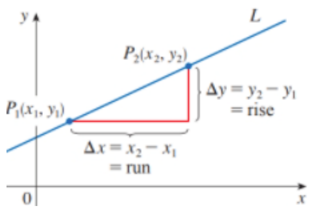
(1)Graph f(x)= $x-|x|$ by first writing it as a piecewise defined function without absolute value bars.



(2) Rewrite the function $f(x)=|2x+3|$ as a piecewise function with no bars.

(3) Rewrite the function $f(x)=|x^2-3x-10|$ as a piecewise function with no bars.

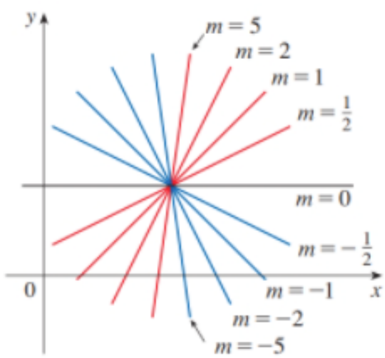
Video Notes: Lines



2 Definitio The **slope** of a nonvertical line that passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.



3 Point-Slope Form of the Equation of a Line An equation of the line passing through the point $P_1(x_1, y_1)$ and having slope m is

$$y - y_1 = m(x - x_1)$$

4 Slope-Intercept Form of the Equation of a Line An equation of the line with slope m and y-intercept b is

$$y = mx + b$$

Example: Find an equation of the line containing the points (4,2) and (-3,6)

6 Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if they have the same slope.
2. Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$; that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

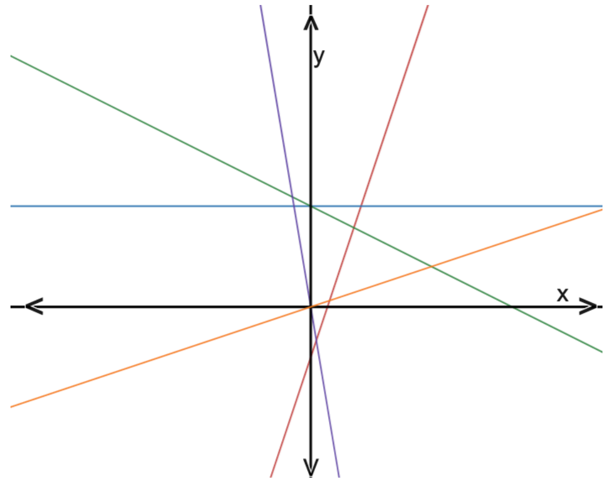
Example: Find an equation of the line containing (4,-2) and perpendicular to 3x-5y=6

Graph Lines

Worksheet: Lines

Match the line color to the slope: (the x and y axes are black)

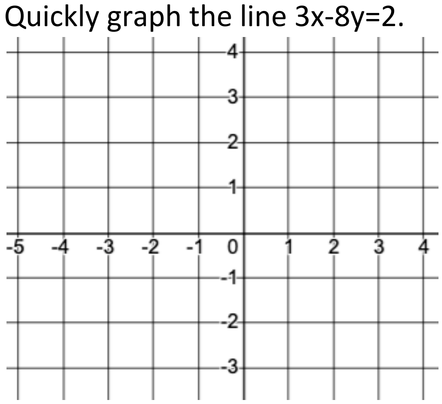
a) m= 1/3 ______________
b) m= 0 ______________
c) m= 3 ______________
d) m= -1/2 ______________
e) m= -6 ______________



Find the equation of the line containing (3,1) and (-1/3, 7)

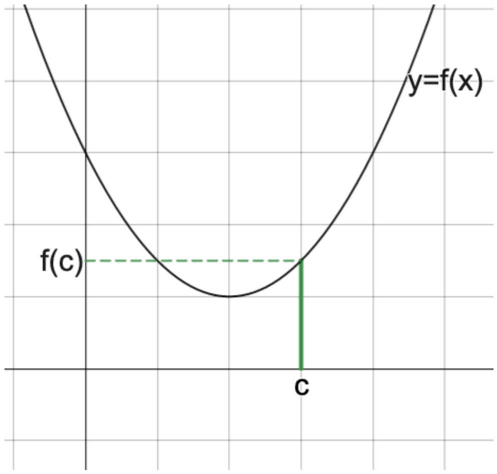
Find the equation of the line containing the x intercept of 2x-7y=3 and perpendicular to 4x+2y=5

Quickly graph the line 3x-8y=2.



Video Notes: Functional Notation and Basic Graphs

Using functional notation when reading a graph



Using functional notation:

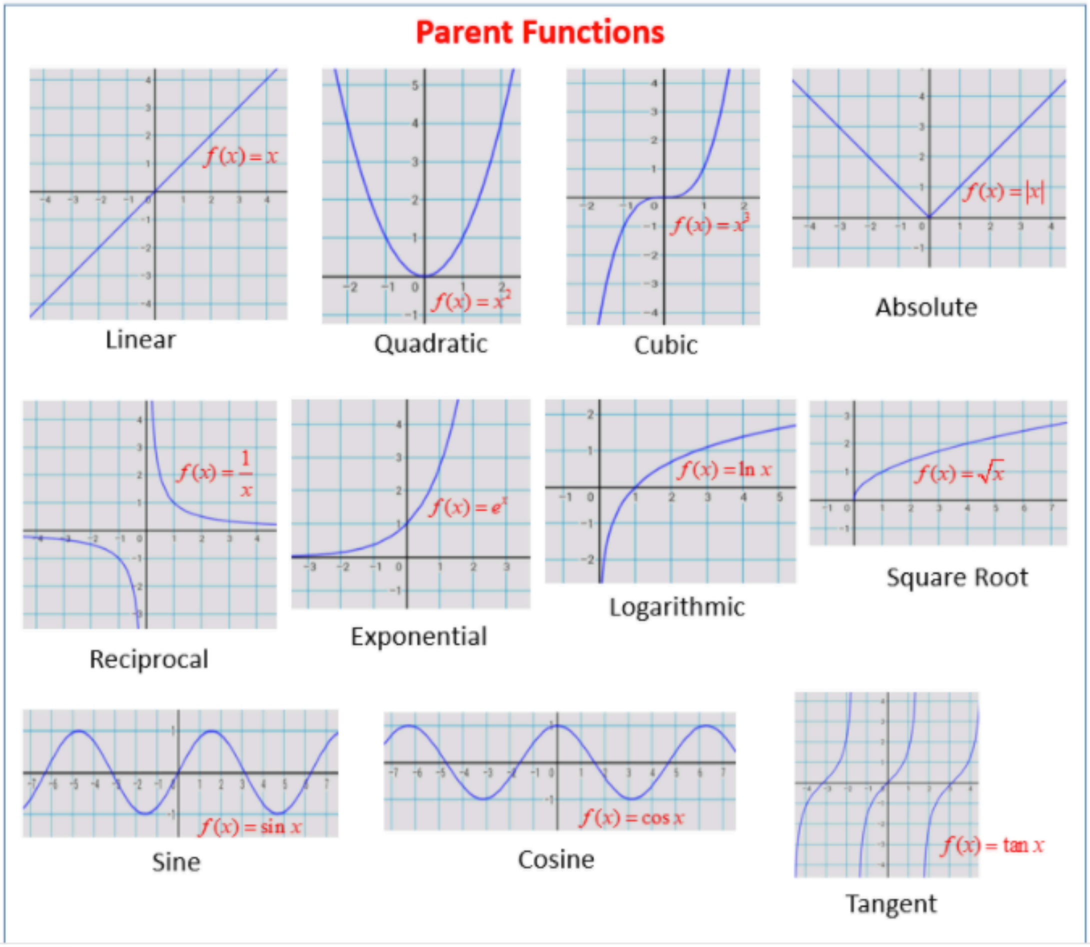
If $y=x^2$, find y when x is 3, -1, and 5

If $f(x)=x^2$, find $f(3)$, $f(-1)$, $f(5)$.

Abstract use of functional notation: If $f(x)= x^2$, find

$f(a)$, $f(x^3)$, $f(2x+3)$, $f(x+h)$ and $\dfrac{f(x+h)-f(x)}{h}$

Basic Parent Function Graphs you should know:

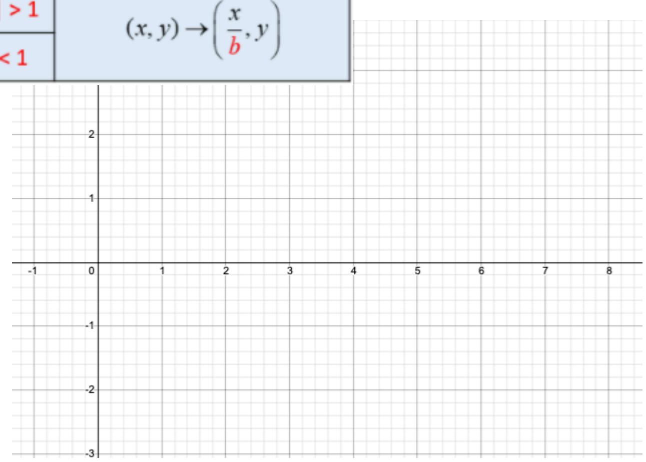


Graphing Transformations.

Transformation Rules for Functions

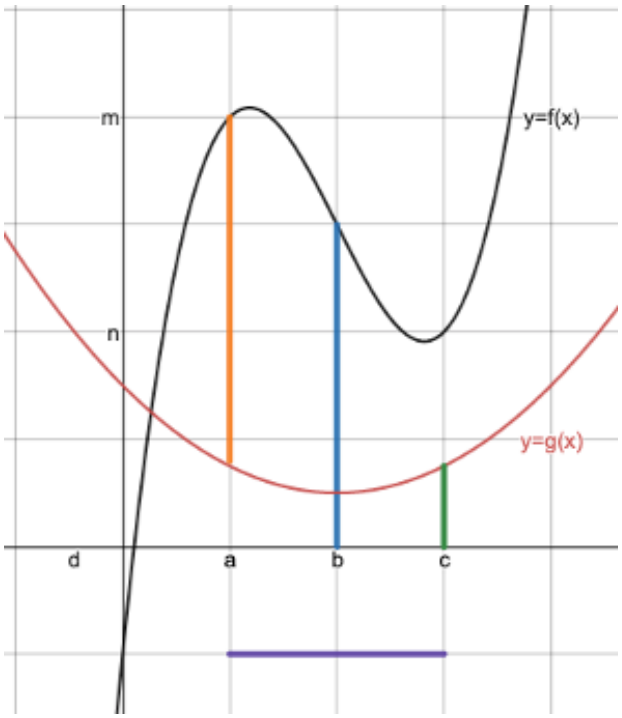
Function Notation	Type of Transformation	Change to Coordinate Point
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) – d	Vertical translation down d units	(x, y) → (x, y – d)
f(x + c)	Horizontal translation left c units	(x, y) → (x – c, y)
f(x – c)	Horizontal translation right c units	(x, y) → (x + c, y)
–f(x)	Reflection over x-axis	(x, y) → (x, –y)
f(–x)	Reflection over y-axis	(x, y) → (–x, y)
af(x)	Vertical stretch for |a| > 1	(x, y) → (x, ay)
	Vertical compression for 0 < |a| < 1	
f(bx)	Horizontal compression for |b| > 1	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
	Horizontal stretch for 0 < |b| < 1	

Example: Graph $f(x) = -2\sqrt{x-4}+1$



Worksheet: Functional Notation

(1) This problem tests your knowledge of reading functional values from graphs. Do not assume any numerical scale. Answers should all be in terms of letters.



What is f(a)?_______________

Find a value of x such that g(x)=n.________________

Using functional notation (not numbers) find the height of each of the blue, green and orange line segments

____

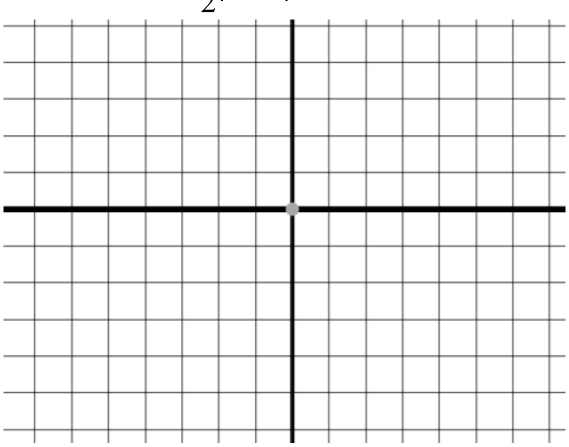
What is the length of the purple segment?

____

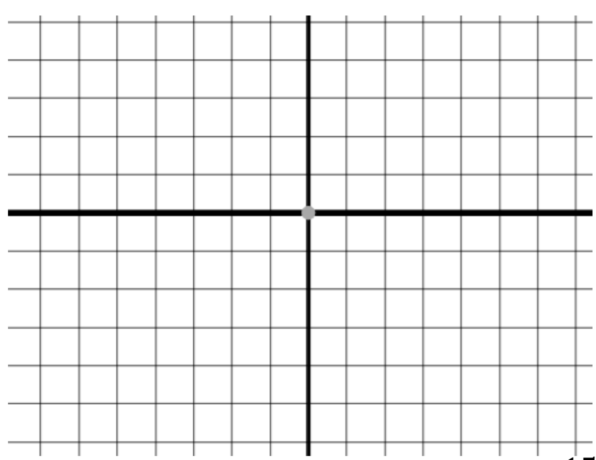
(2) Given $g(x)=\frac{1}{x^3}$, find $g(a)$, $g\left(\frac{2}{x}\right)$, $g(3x+1)$, $\frac{g(x+h)-g(x)}{h}$

(3) Quickly sketch the graph of (you should not need a big table of points)

(a) $f(x)=\frac{1}{2}|x+3|-4$



(b) $g(x)=4-2\cos x$

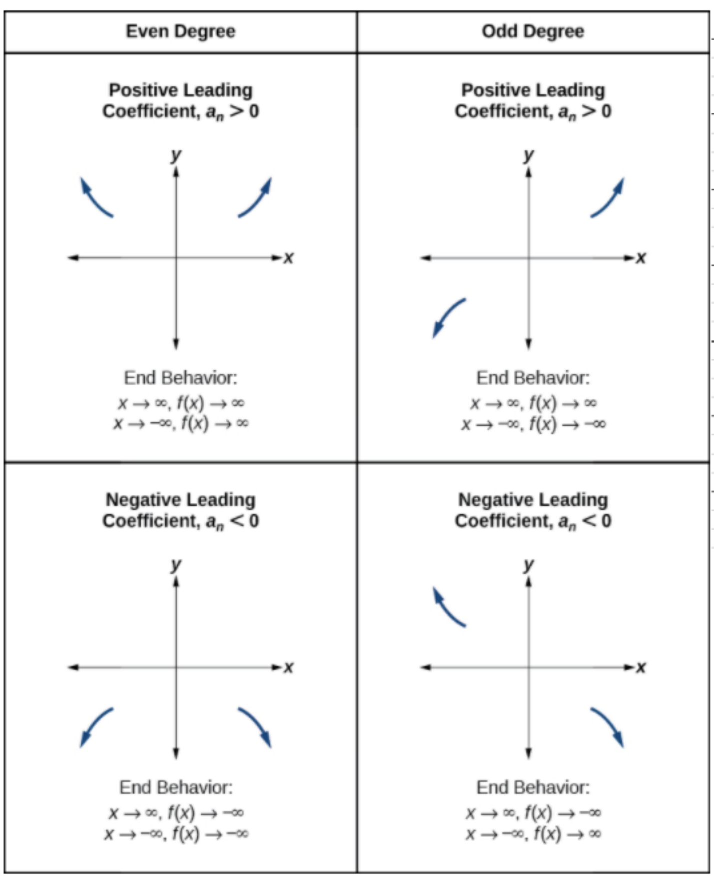


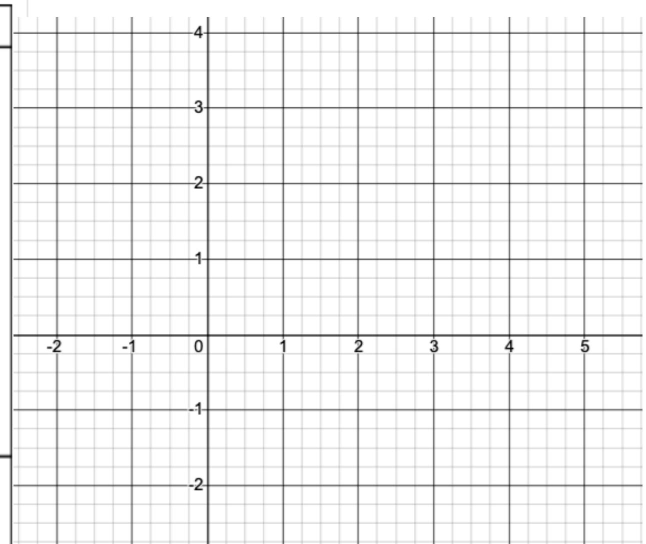
Video Notes: Graphing Polynomials and Rational Functions – the Basics

Polynomial:

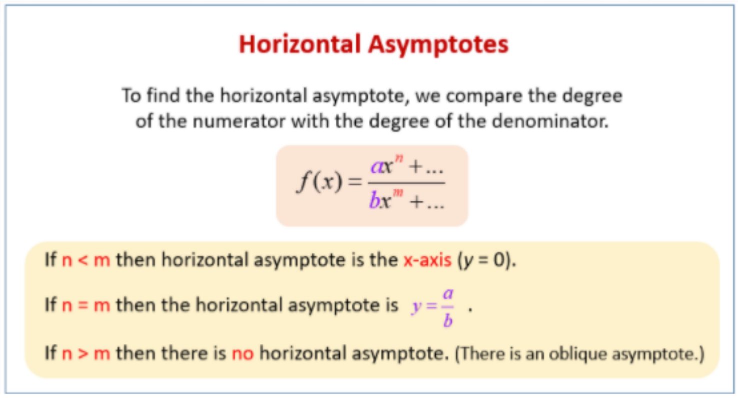
- Smooth, rolling graphs.
- For polynomial of degree n there are at most n x intercepts and at most n-1 turns.
- Find and graph y intercept.
- Find and graph x-intercepts if possible.
- Determine end Behavior:

Example: Graph $f(x)=\frac{1}{3}(x-3)^2(2x+1)$

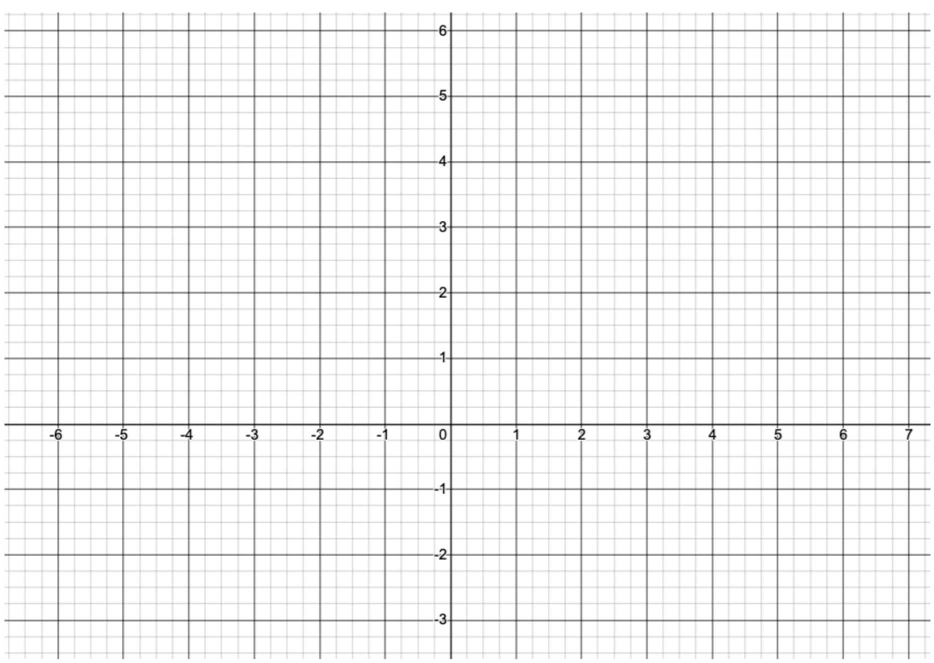




Graphing a Rational Function:

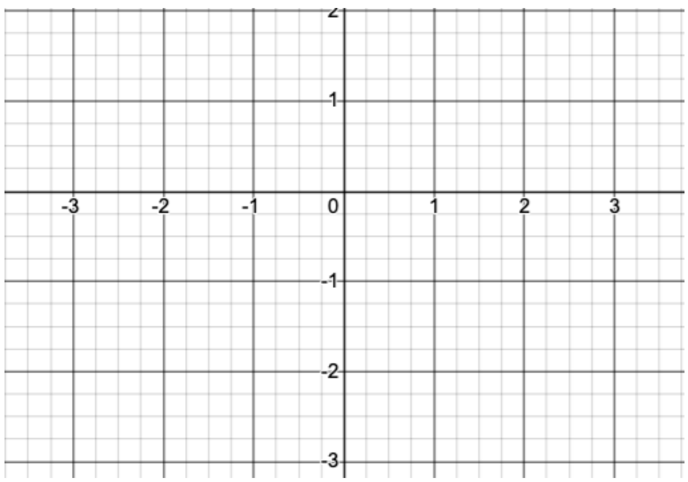
- Factor numerator and denominator and reduce fraction if possible.
- Find y intercepts,
- Find x intercepts if possible,
- Find Vertical Asymptotes if any and consider approach up or down.
- Find Horizontal Asymptotes if any
  - 

Example: Graph $f(x)=\dfrac{2x^2-4x}{(x-1)(x+2)}$



Worksheet: Graphing Polynomials and Rational Functions – the Basics

Sketch the graph of $f(x) = -\frac{x}{2}(x+2)^2(x-1)$



Sketch the graph of $f(x) = \frac{2x}{x^2-4}$